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**ENGINEERING ANALYSIS**

**SUMMER 2015**

**HOMEWORK #3 PROBLEM 7**

Solve the one dimensional heat equation below with a time-dependent heat source and non-homogeneous boundary conditions:

Boundary Conditions:

Initial Conditions:

We need to make the boundary conditions homogeneous first, we will introduce the following new function,

And we will use this in our displacement function v(x,t):

Notice,

So we have achieved our new homogeneous boundary conditions and a new initial condition for v(x,t) satisfying,

The expansion of v(x,t) for the homogeneous differential equation: is,

For the non-homogeneous problem we will utilize variation of parameters to get a similar eigenfunction expansion,

Let us now use this in our displaced differential equation for v(x,t):

Substitute these above two lines into the differential equation we get,

Or

**Part a) Let us consider the case when ,**

Notice that we are ‘filtering’ out only two terms from the series on the left when n = 3 and n = 4, thus;

Or in a more general format,

We have three differential equations implied and for each we will need to solve the homogeneous problem and non-homogeneous problem

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The coefficients can be found,

Thus,

Where,

Our solution to the original differential equation will be,

**Part b) Let us consider the case when ,**

Again, we are ‘filtering’ out our solution when n = 5, hence,

Thus we have two differential equations,

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Where,

Thus,

Where,

Thus,